Details on the Sample Size Calculator for the Cochran-Armitage Test

The Cochran-Armitage trend test in proportions involves an ordered set of groups for which we test for a linear trend in the proportions responding as we go across the ordering. The linear trend in the probabilities of response is given by the expression:

\[ p_i = a + bd_i, \]

where we test for a non-zero slope relating the probabilities to the numeric assigned to each of the ordered groups. For details on the test statistic see Agresti (2002) and see similar details as well as SAS implementation of the test at this link:


Nam (1987) notes that the test statistic is equivalent to using a linear logistic model instead and testing for the slope associated with ordered levels in this logistic model. Nam evaluates the sample size using this logistic model.

Using the same notation as in Nam, we define \( p_i \) as the probability of response, \( q_i \) as the probability of non-response, \( d_i \) as the ordered numeric assigned to a group (typically the actual dose when looking at dose levels or ordinarily ordered as equal to the subscript \( i \)) and \( r_i \) as the multiple of the sample size in a group \( n_i \) to that in the control \( n_0 \) for \( i = 0 \) to \( (k-1) \) (\( k \) groups including control). Further \( \bar{d} \) is the average of the \( d_i \)'s, \( z_{1-\alpha} \) is the \((1-\alpha)\)th quantile the standard normal distribution, \( z_{1-\beta} \) is the \((1-\beta)\)th quantile of the standard normal distribution, \( \Delta \) is the difference between successive \( d_i \)'s and \( p = \left( \sum r_i p_i \right) \left( \sum r_i \right) \). Nam (1987) provide the following expression for the continuity corrected sample size for the Cochran-Armitage Trend test in the control arm

\[ n_0^* = \left( n_0^* / 4 \right) \left[ 1 + \sqrt{1 + 2\Delta / (An_0^*)} \right]^2, \]

where \( n_0^* \) is the sample size in the control arm without the continuity correction and is give by

\[ n_0^* = \left[ z_{1-\alpha} \sqrt{pq \left( \sum r_i (d_i - \bar{d})^2 \right) + z_{1-\beta} \sqrt{\sum r_i p_i q_i (d_i - \bar{d})^2} } \right] / A^2, \]

for \( A = \sum r_i p_i (d_i - \bar{d}) \).

References:


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